EFFICIENCY COMPARISON OF REGULARIZED G-N AND NCGM ALGORITHMS IN CASE OF CRACKS AND FLAWS IDENTIFICATION IN CONDUCTIVE MATERIALS

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***Abstract*: In the work the multi-frequency method for crack and flaw recognition on the basis of sensitivity analysis in Finite Elements Method (FEM) is shown. This inverse job consists in iterative Gauss-Newton (G-N) algorithm with either Truncated Singular Value Decomposition (TSVD) or Conjugate Gradient Method on the Normal Equation (CGLR) as a regularization tool. Other proposed method for solution of inverse problem is the Nonlinear Conjugate Gradient Method (NCGM). In case of real measurement data the Generalized Cross Validation (CGV) method was applied as a stop criterion of the recognition processes. Proposed methods suit as well surface cracks, as flaws inside conducting materials.**

***Keywords*: nonlinear ill-posed problems, multi-frequency sensitivity analysis, regularization with GCV**

I. INTRODUCTION

Our objective is to solve the inverse problem of crack shape recognition arising in eddy-current method of non-destructive testing. To solve the over-determined system of equations the regularization parameter has to be chosen. There are different ways to obtain this quantity. In this paper we propose GCV technique. This method could be also applied to the estimation of noise parameters modeling measurement errors.

II. OPTIMIZATION ALGORITHM

A. Optimization task

$$\min_{p^*} F(\boldsymbol{p}^*, \ell^*) = \sum_k \min_{\Delta p^k} \left\| \boldsymbol{S}^k \Delta \boldsymbol{p}(\ell^k)^k - \boldsymbol{r}^k \right\|_2^2 + \left\| \boldsymbol{I} \Delta \boldsymbol{p}(\ell^k)^k \right\|_2^2 \quad (1)$$

where $\boldsymbol{S}$ – the $m \times n$ sensitivity matrix, ℓ^k – truncation parameter, $\boldsymbol{I}$ – the identity matrix, , $\boldsymbol{r}$ – the known vector (the difference between the measurements and data calculated by FEM).

B. Multi-frequency sensitivity analysis

For effective solution of such inverse job in iterative manner the quasi-Hessian and gradient of assumed goal function have to be evaluated. We apply for this purpose multi-frequency sensitivity analysis of electromagnetic field quantities versus such material parameter, as conductivity, or versus geometrical parameter, as the air-conductor border [1].

C. Regularization tools

The GCV-technique was applied to determinate the truncation parameter, which depends on noise level ε. This method suggests choosing as a regularization parameter the index ℓ, that minimizes the GCV function [3]:

$$GCV(\ell) = \frac{\left\| \boldsymbol{S} \Delta \boldsymbol{p}_\ell - \boldsymbol{r} \right\|_2^2}{tr(\boldsymbol{I} - \boldsymbol{S}\boldsymbol{S}_\ell^\dagger)^2}, \quad \ell = 1, 2 \ldots . N = rank(\boldsymbol{S}) \quad (2)$$

D. Regularized Gauss-Newton method

For every step k of the identification task the vector of corrections $\Delta \boldsymbol{p}$ is calculated from equation:

$$\boldsymbol{p}(\ell)^{k+1} = \boldsymbol{p}(\ell)^k - \left[\left[\boldsymbol{S}^k \right]^T \boldsymbol{S}^k \right]^{-1} \left[\boldsymbol{S}^k \right]^T \boldsymbol{r}^k \quad (3)$$

E. Nonlinear conjugate gradient method (NCGM)

In case of NCGM the classic formula could be described as:

$$\begin{aligned} p^{k+1} &= p^k - \beta^k d^k \\ d^k &= -\nabla F(p^k) + \alpha^k d^k \qquad k = 1, 2, \ldots, N \end{aligned} \quad (4)$$

where β, α are coefficients responsible for conjugate search directions and line search, respectively.

III. NUMERICAL EXAMPLES

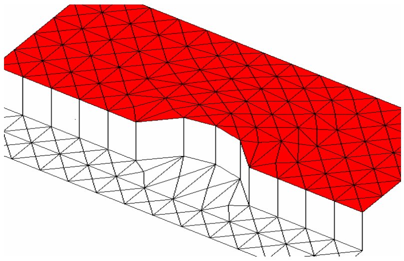

Figure 1: Surface crack after 6th iteration.

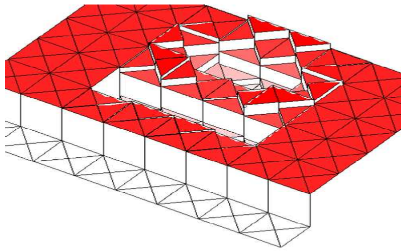

Figure 2: Conductivity distribution after 7th iteration.

IV. CONCLUSIONS

The numerical experiments with different number of excitation frequencies, with many coil positions and finite element meshes show the practical uniqueness of obtained solutions. Now, the work is continued for 3D models.

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