Adjoint Models in the Time Domain Sensitivity Analysis Utilizing FEM for the Identification of Material Conductivity Distribution

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Abstract — The paper deals with numerical aspects of the sensitivity analysis in time domain. The dependence of adjoint model excitation from the way of magnetic field measurement is discussed. The structure of adjoint model bases on a version of Tellegen's theorem for electromagnetic field theory. Proposed method finds application in inverse tasks of conductive material testing using eddy-currents.

I. INTRODUCTION

Sensitivity evaluation utilizing adjoint systems is very effective. In this work the algorithms for time-domain 2D sensitivity analysis versus conductivity values in separate finite elements are presented. Time-domain analysis offers the possibility, to choose a convenient shape of excitation in adjoint system. The time- and space-discretization bases on first order finite elements. The environment is assumed to be linear and isotropic. To verify achieved results the direct method of sensitivity analysis and incremental model has been used[1].

II. TRANSIENT MAGNETIC FIELD ANALYSIS UTILIZING FEM

For the magnetic field transient analysis the typical finite element algorithm is used, utilizing generalized time stepping scheme *theta* [2]. Dividing the time range (0, T) into n time steps of the length $\Delta t = T/n$, the differential scheme can be written as follows:

$$\left[\Theta[\mathbf{K}]+\frac{1}{\Delta t}[\mathbf{M}]\right]\{\mathbf{U}_i\}= =\left\{\left(\frac{1}{\Delta t}[\mathbf{M}]-(1-\Theta)[\mathbf{K}]\right)\{\mathbf{U}_{i-1}\}+(1-\Theta)\{\mathbf{i}_i\}+\Theta\{\mathbf{i}_{i-1}\}\right\}, \tag{1}$$

where [**K**] and [**M**] are the stiffness and mass matrices of finite elements containing the material parameters and geometric properties of the simulated model, $\{\mathbf{U}_i\}$ is the vector of the desired node values (modified magnetic vector potentials of nodes) and $\{\mathbf{i}_i\}$ is the discretized excitation for time steps $i\cdot\Delta t$, with $i = 1...n$ and the parameter Θ determines the time stepping scheme. $\{\mathbf{U}_0\}$ is the initial condition vector. In most cases of field penetration into conducting region, this vector should be set to zero. Knowledge of modified magnetic vector potential distribution allows calculating voltage induced in measurement coil in time domain.

III. SENSITIVITY ANALYSIS

The algorithm bases on Tellegen's theory, commonly applied in circuit analysis [3]. Conception of application of this method to electromagnetic field theory for the frequency domain was shown in the work [4], and the extension for time-domain sensitivity analysis was described in [1]. The Tellegen's sensitivity equation may be derived from Lorenz reciprocity theorem. Two systems have to be analyzed, the adjoint one has the same topology and material parameters, it differs from original only with the excitation. Both are analyzed on the same area Ω, but for different times t and τ. The time τ is reverse to t, it means $\tau = \xi - t$, where ξ denotes the time, while the sensitivity is evaluated.

For the tasks of electric field sensitivity versus electric conductivity γ, the sensitivity equation simplifies to:

$$\int_0^{\xi}\iint_{\Omega} J_o^{+}(\tau)\cdot\delta E(t)\,\mathrm{d}\Omega\,\mathrm{d}t=\int_0^{\xi}\iint_{\Omega} E^{+}(\tau)\cdot\delta\gamma\cdot E(\tau)\,\mathrm{d}\Omega\,\mathrm{d}t \tag{2}$$

where E is the only non-zero component of electric intensity vector, perpendicular to the plane of analysis and J_o excitation current density. The variables denotes with ($^+$) relate to the adjoint system, other one to original. The sensitivity equation shows the changes in δE caused by conductivity variation $\delta\gamma$.

IV. TEST MODEL

The simple model of non-destructive testing probe is shown in Fig. 1. It consists of exciting and measurement coils placed over a conducting plate.

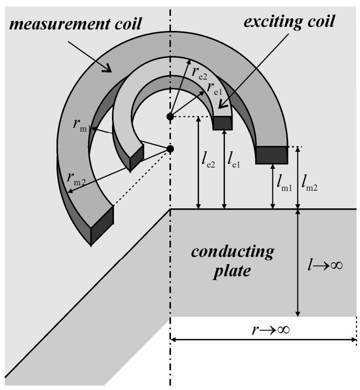

Fig. 1. Coils over conducting plate

The problem is assumed to be axial symmetric and can be analyzed using 2-D formulation (Fig. 2a). The Dirichlet boundary condition is given for all boundaries of model. The finite element mesh for this model consists of 960 elements and 513 nodes (Fig. 2b).

Using Fourier transform the voltage induced in measurement coil can be analytically calculated. The analytical solution in the time domain for this test model is available in [5].

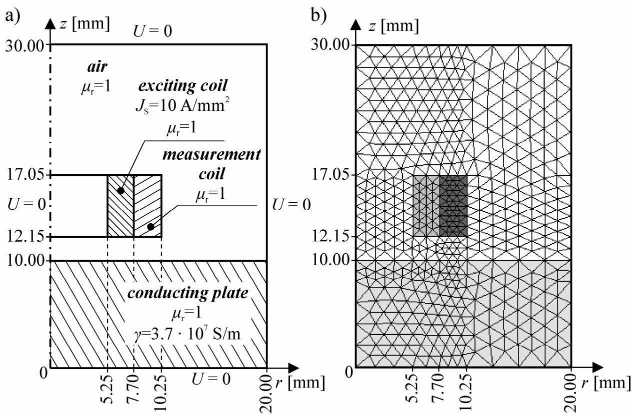

Fig. 2. a) Analyzed model, b) finite element mesh

V. ADJOINT MODELS

The excitation of adjoint model depends on the manner, how the magnetic field over the plate has been measured:

A. magnetic field measurement using single loop-coil.

Adjoint model should be excited with the current driven to the node corresponding to coil position (Fig 3a).

B. magnetic field measurement using coaxial coil with finite cross-section.

No current exists in area of excitation coil, excitation current displaced uniformly on area of measurement coil (Fig. 3b).

C. impedance measurement of exciting coil.

Exciting coil takes over the role of measurement coil. The structure of adjoint model is identical to that of original (Fig 3c), only the excitation impulse shape is different.

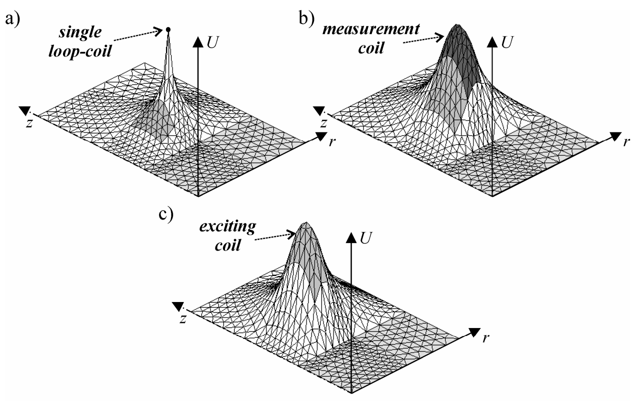

Fig. 3. Modified vector potential *U* distributions on adjoint models.

VI. EXCITATION SHAPE OF ADJOINT MODEL

In the case of transient sensitivity analysis the shape of excitation has to be chosen [6]. The right choice of adjoint model excitation leads to simplification of the left hand side of equation (2). We propose application of unit step current impulse: $i_0^+(\tau) = 1\text{A}$ for $\tau > 0$ and $i_0^+(\tau) = 0$ otherwise. The assumed excitation is not realizable physically, it acts only in virtual, adjoint model.

The time-discretization scheme forces a very special form of discretized unit-step. The excitation remains linear inside finite elements and continuous in their nodes. The numerical representation of unit-step impulse is as shown in Fig. 4.

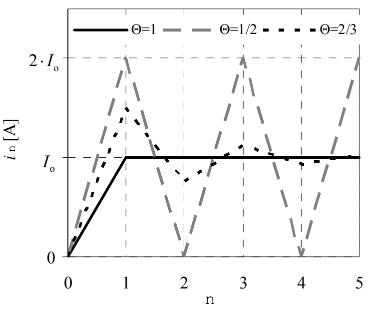

Fig. 4. The discretized excitation of adjoint model for first five time steps and different values of *Θ*

The shape closely relates to parameter Θ assumed for the stepping scheme (1). The amplitude of the excitation could be assumed $I_o = 1\text{A}$.

VII. CONCLUSIONS

The described adjoint models are successfully used by the authors for solution of inverse problems concerning recognition of conductivity distribution and utilizing deterministic Gauss-Newton optimization algorithm [6]. The gradient information necessary for this method was determined from sensitivity analysis in the time domain.

VIII. REFERENCES

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